GCSE Maths – Geometry and Measures

Surface Area of 3D Shapes

Notes

WORKSHEET

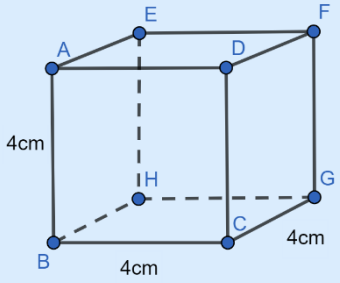
Surface Area of 3D Shapes

The surface area of a 3D shape is the **total area covering** the outside of the shape. Since it is a measure of **area**, the units are **units²**, such as cm^2 or km^2.

Surface area of cubes and cuboids

To find the surface area of a **cube**, we **add together the area of each side**. As each side is a **square** of equal area, we can simply multiply the area of **one face by 6**.

Example: Find the surface area of the following cube.



1. Find the area of one face of the cube.

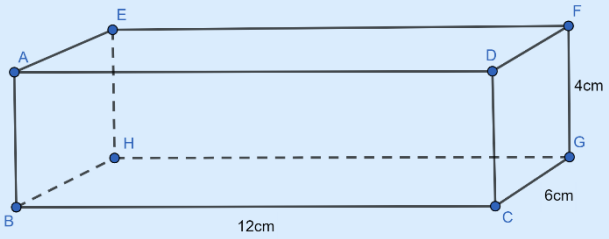
$$Area = 4 \times 4 = 16 \text{ cm}^2$$

2. *Find the total surface area of the cube by multiplying the area of one face by* 6.

$$Surface\ area = 16 \text{ cm}^2 \times 6 = \mathbf{96\ cm^2}$$

For **cuboids**, we need to find the area of each face.
The **opposite faces** of a cuboid are **equal**. This means that we only need to calculate the area of 3 sides, as we can **double** the areas of each pair we find.

Example: Find the surface area of the following cuboid.



1. Find the area of each of the different faces we can see.

$$12 \times 4 = 48 \text{ cm}^2$$
$$12 \times 6 = 72 \text{ cm}^2$$
$$4 \times 6 = 24 \text{ cm}^2$$

2. Find the total surface area using the values found.

We have two of each of these different faces. So, we double the above values:

$$48 \times 2 = 96 \text{ cm}^2$$
$$72 \times 2 = 144 \text{ cm}^2$$
$$24 \times 2 = 48 \text{ cm}^2$$

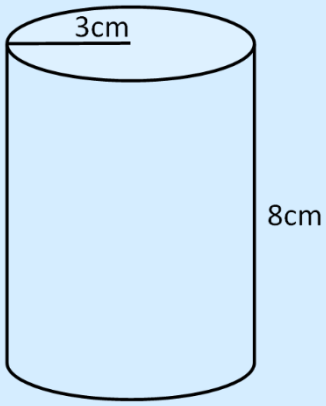
$$Surface\ area = 96 \text{ cm}^2 + 144 \text{ cm}^2 + 48 \text{ cm}^2 = \mathbf{288\ cm^2}$$

Surface area of cylinders

To find the surface area of a **cylinder**, we must calculate the area of the **two circular faces** and the area of the **curved face** connecting the ends.

The curved face in the middle is essentially a **rectangle**. You can picture this by imagining if you sliced down a cylinder and flattened it out, the curved face would lay flat into a rectangle. This rectangle has length equal to the length of the cylinder itself, and its width is the **circumference** of the circular face.

Example: Find the surface area of this cylinder, giving your answer to 2 decimal places.



1. Find the area of the circular faces using the formula for the area of a circle:

$$Area = \pi \times 3^2 = 9\pi = 28.274 \text{ cm}^2$$

Since there are two faces, double this value to find the total surface area of the circular faces:

$$Area = 56.549 \text{ cm}^2$$

2. Find the area of the curved face between the two circular faces.

Find the area of the curved face by multiplying the circumference of the circular face by the length of the cylinder:

$$Area\ of\ curved\ face = \pi \times 6 = 6\pi = 18.850 \text{ cm}^2$$

3. *Find the total surface area of the cylinder.*

Add together the area of the circular faces and the middle, curved face:

$$Total\ surface\ area = 56.549 \text{ cm}^2 + 18.850 \text{ cm}^2 = \mathbf{75.40\ cm^2}\ (2\ d.p.)$$

Surface area of pyramids

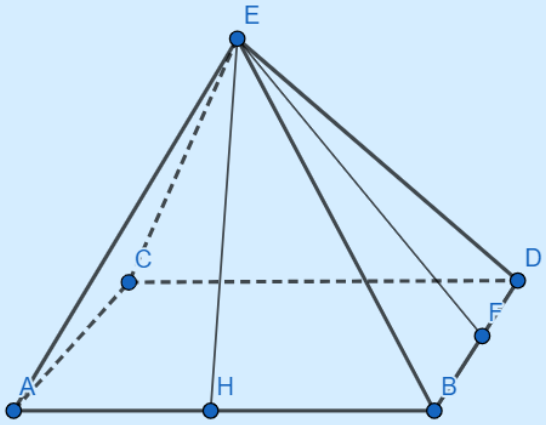
To find the surface area of pyramids, we need to find the surface area of the **base** and the area of the **triangular faces** making up the sides of the pyramid.
Note, the base could be a **square**, **triangle** or **rectangle**.

All triangular faces will have the same **height** but might not have the same base (if it is a rectangle-based pyramid, for example).

Example: Find the surface area of this rectangle-based pyramid, where:

$$AC = 2.6 \text{ cm}$$
$$AB = 8.3 \text{ cm}$$
$$EF = 7 \text{ cm}$$
$$EH = 5.7 \text{ cm}$$

Give your answer to 3 significant figures.



1. Find the surface area of the base.

   The base here is a rectangle, so we multiply the length of the base by the width of the base:

   $$Base\ area = 8.3 \times 2.6 = 21.58 \text{ cm}^2$$

2. Find the area of the triangular faces using the formula for the area of a triangle.

   The opposite triangular faces will be identical because they have the same base and height.

   $$Area\ of\ triangular\ face = \frac{triangle\ base \times triangle\ height}{2}$$

   $$Area\ of\ left\ and\ right\ triangular\ faces = \frac{2.6 \times 7}{2} = 9.1 \text{ cm}^2$$

   $$Area\ of\ front\ and\ back\ triangular\ faces = \frac{8.3 \times 7}{2} = 23.66 \text{ cm}^2$$

3. Double the area of each of the triangular faces found (because the opposite face is the same) to find the total surface area of all triangular faces. Add it to the base area to find the total surface area.

   $$Triangular\ faces\ area = (9.1 \times 2) + (23.66 \times 2) = 65.52 \text{ cm}^2$$

   $$Total\ surface\ area = 65.52 + 21.58 = \mathbf{87.1\ cm^2}$$

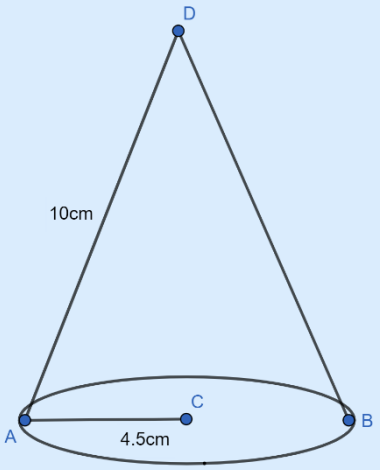
Surface area of cones

A cone consists of a **circular face** at the base and a **curved face.**

To find the surface area of the circular face, use the formula for the area of a circle, $\boldsymbol{A = \pi r^2}$.

To find the surface area of the curved face, we use the formula $\boldsymbol{A = \pi \times r \times l}$, where *l* is the sloped length of the cone. It is important to not confuse the sloped length l with the perpendicular height h.

Example: Find the surface area of the following cone, giving your answer to 2 decimal places.



1. Find the area of the circular face at the base.

$$Base\ area = \pi \times 4.5^2 = 63.617\ \text{cm}^2$$

2. Find the area of the curved face using the formula $A = \pi \times r \times l$, where r is the radius of the base and l is the sloped length.

$$Curved\ face\ area = \pi \times 4.5 \times 10 = 141.37\ \text{cm}^2$$

3. Find the total surface area.

Add the previous areas obtained to find the total surface area:

$$Total\ surface\ area = \mathbf{234.99\ cm^2}$$

Surface area of spheres

The formula for the surface area of a sphere is given by the following formula:

$$Surface\ area\ of\ a\ sphere = 4\pi r^2$$

Example: What is the surface area of a sphere with radius 14 cm?

Use the formula for the surface area of a sphere:

$$Surface\ area\ of\ a\ sphere = 4\pi r^2 = 4 \times \pi \times 14^2 = \mathbf{2463.01\ cm^2}$$

Surface area of composite shapes

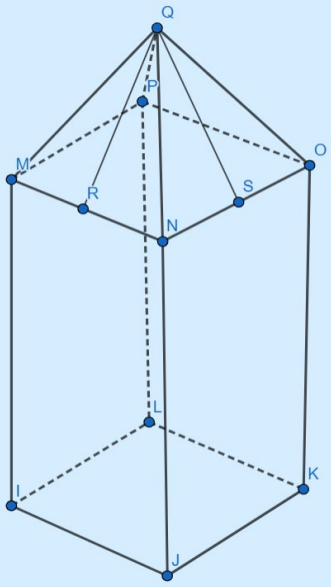
Composite 3D shapes are simply shapes made up of **two or more different 3D shapes**, such as a cylinder and a sphere.

To find the surface area of a composite shape, consider which sides are **not facing outwards**. Work out the surface area of the sides that are on the **outside**.

Example: Calculate the total surface area of the following composite shape, where:

$$\text{MI} = 6 \text{ cm}$$
$$\text{JK} = 3.5 \text{ cm}$$
$$\text{IJ} = 3 \text{ cm}$$
$$\text{QR} = 4 \text{ cm}$$
$$\text{QS} = 3.9 \text{ cm}$$

Give your answer to 3 significant figures.



1. First, consider which faces are on the outside - these are the ones that we need to calculate the area of.

   All sides of the cuboid except the top; all sides of the pyramid except the base.

2. Find the surface area of the required faces.

   Cuboid:

   $$\boldsymbol{Bottom\ face}:$$
   $$3 \times 3.5 = 10.5 \text{ cm}^2$$

   $$\boldsymbol{Side\ faces}:$$
   $$Area\ of\ side\ faces\ with\ base\ 3 \text{ cm} = 3 \times 6 = 18 \text{ cm}^2$$
   $$Area\ of\ side\ faces\ with\ base\ 3.5 \text{ cm} = 3.5 \times 6 = 21 \text{ cm}^2$$

   $$Total = 10.5 + 2(18) + 2(21) = 88.5 \text{ cm}^2$$

   Pyramid:

   $$\boldsymbol{Triangular\ faces}:$$
   $$Area\ of\ triangular\ faces\ with\ base\ 3 \text{ cm} = \frac{3 \times 4}{2} = 6 \text{ cm}^2$$
   $$Area\ of\ triangular\ faces\ with\ base\ 3.5 \text{ cm} = \frac{3.5 \times 3.9}{2} = 6.825 \text{ cm}^2$$

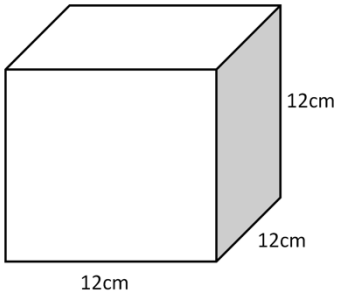
   $$Total = 2(6) + 2(6.825) = 25.65 \text{ cm}^2$$

3. Find the total surface area.

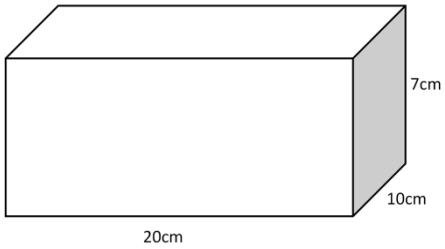
$$Surface\ area\ of\ composite\ shape = 88.5 + 25.65 = 114.15 \text{ cm}^2 = \mathbf{114 \text{ cm}^2}\ (3\ s.f.)$$

Surface Area of 3D Shapes – Practice Questions

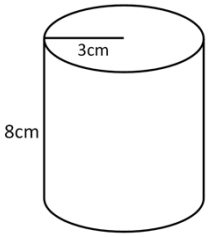
1. Calculate the surface area of this cube.



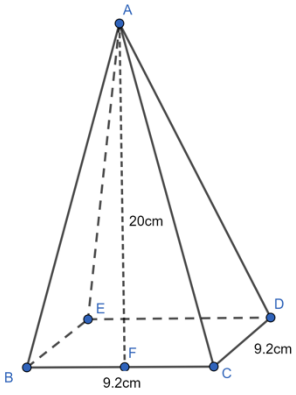
2. Calculate the surface area of this cuboid.



3. Calculate the surface are of this cylinder.



4. Calculate the surface area of this cone.



Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.